SPECTRAL PROPERTIES OF PHOTON PAIRS GENERATED IN PROCESS OF PARAMETRIC DOWN CONVERSION

PIOTR KOLENDERSKI, WOJCIECH WASILEWSKI AND KONRAD BANASZEK

Institute of Physics, Nicolaus Copernicus University ,
ul.Grudziadzka 5, 87-100 Torun, Poland

ABSTRACT

We present a numerical method for optimizing sources of biphotons and heralded single photons based on spontaneous parametric down-conversion (SPDC) coupled into single-mode fibers (SMF) [1], shown in Fig. 1. Our approach enables us to calculate efficiently with low computation power the complete wave function of the generated state in the spectral domain. This provides a tool to analyze a number of performance criteria such as source brightness and spectral correlations within generated pairs, which has been applied to both type I [2] (see Fig. 2) and type II [3] (see Fig. 3) configurations. By avoiding some of crude approximations used previously [4], our model provides reliable results in a broader range of parameters of the experimental setup.

INTRODUCTION

We consider a $\chi^{(2)}$ nonlinear crystal of thickness L pumped by a pulsed gaussian beam of pulse duration τ_p and beam waist w_p. Generated photons are coupled into SMF. Our objects of interest are the spectral wave function $\psi(\omega_s, \omega_i)$ of the photon pair and the reduced density matrix $\rho(\omega, \omega')$ of single photons for spatial modes defined by SMF.

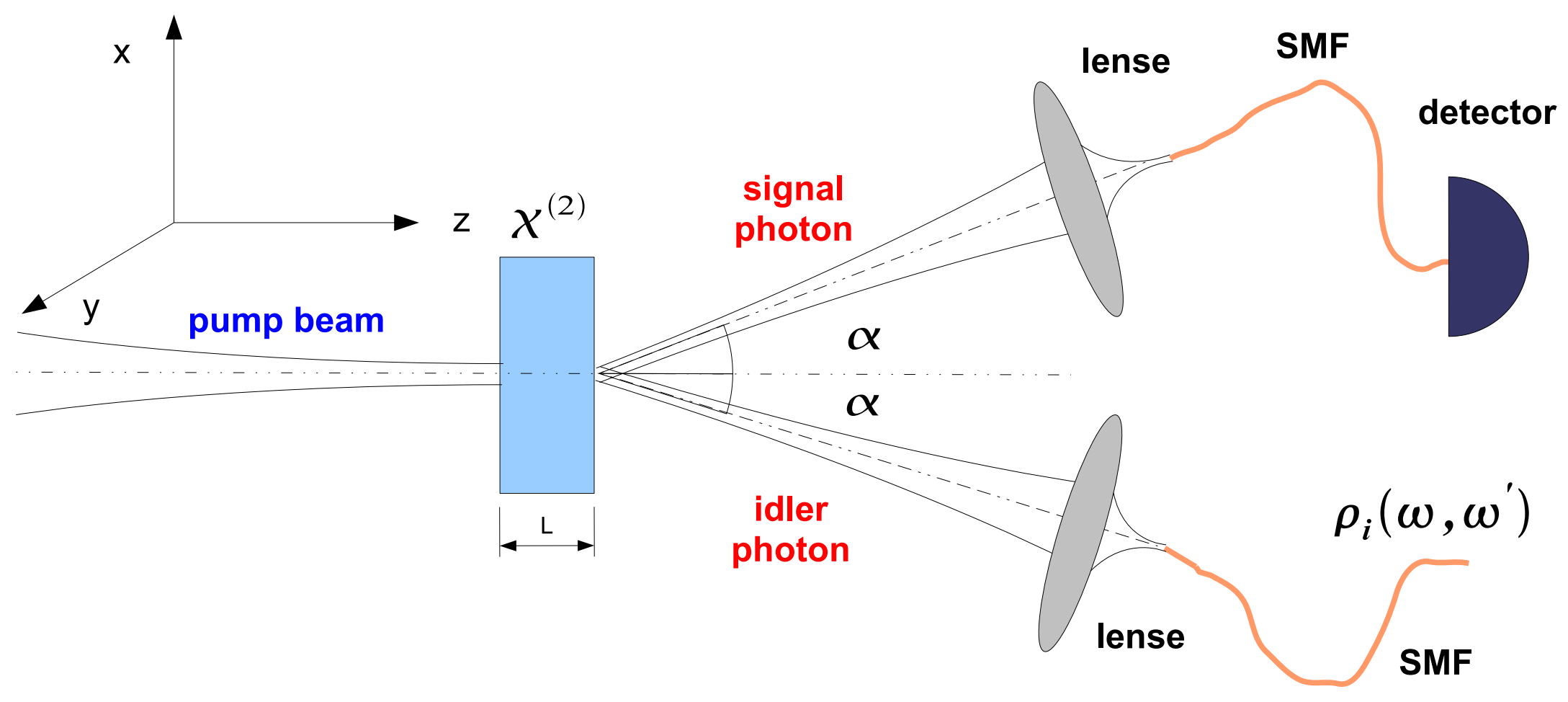

FIGURE 1: Geometry of the photon source under consideration. For a type-II process, coupling lenses are additionally preceded by birefringent compensating crystals and polarizers.

APPROACH

Steps:

- We parameterize the biphotons with their frequencies ω_s, ω_i and transverse wave vectors $\mathbf{k}_{s\perp}, \mathbf{k}_{i\perp}$, where s and i correspond to the signal and the idler fields [5].
- The complete biphoton wave function parameterized with $(\omega_s, \mathbf{k}_{s\perp}; \omega_i, \mathbf{k}_{i\perp})$ involves an integral over the volume of the crystal of triple products of plane waves $\exp[i(k_{lz}z + \mathbf{k}_{l\perp}\mathbf{r}_\perp)]$, $l = p, s, i$, where p corresponds to pump pulse.
- Integral over $\mathbf{r}_\perp$ yields conservation of the transverse momentum: $\mathbf{k}_{p\perp} = \mathbf{k}_{s\perp} + \mathbf{k}_{i\perp}$.
- Analogously, integration over time gives $\omega_p = \omega_s + \omega_i$.
- The biphoton wave function inside SMF is obtained by performing projection onto spatial modes determined by the fibers.
- Longitudinal spatial frequencies k_{lz} are functions of ω_s, ω_i and $\mathbf{k}_{s\perp}, \mathbf{k}_{i\perp}$.
- The relevant ranges of $\mathbf{k}_{s\perp}$ and $\mathbf{k}_{i\perp}$ are restricted by the fiber modes.
- This justifies quadratic expansion of k_{lz} in $\mathbf{k}_{s\perp}$ and $\mathbf{k}_{i\perp}$ around central collection directions; the dependence on ω_s and ω_i is kept exact, retaining all the orders of dispersion.
- Assuming the Gaussian shape of fiber and pump modes, all the integrals in transverse xy degrees of freedom can be evaluated analytically yielding the two-photon wave function $\psi_f(z; , \omega_s, \omega_i)$ generated by a slice $(z, z + dz)$ of the crystal [6].
- The total generated wave function is thus given by a single integral over z:

$$\psi_f(\omega_s, \omega_i) = \int_{-L/2}^{L/2} dz\, \psi_f(z; , \omega_s, \omega_i)$$

which can be efficiently evaluated by numerical means.

PERFORMANCE CRITERIA

- We define the source brightness as the photon pair generation rate $p_2 = \langle \psi_f | \psi_f \rangle$.
- Exploiting Schmidt decomposition [7, 8] we may rewrite the biphoton wave function in the following way:

$$\psi(\omega_s, \omega_i) = \mathcal{N} \sum_{n=0}^{\infty} \sqrt{\xi_n} \phi_n(\omega_s)\, \phi_n(\omega_i)$$

where $\{\phi_n(\omega)\}$ is a set of orthogonal modes, and $\{\xi_n\}$ are probabilities of generating the nth pair of Schmidt modes. After signal photon detection the idler photon is described by a conditional density matrix of the form:

$$\rho_i(\omega, \omega') = \sum_n \xi_n \phi_n(\omega)\, \phi_n^*(\omega')$$

The probability ξ_0 of the dominant Schmidt mode $\phi_0(\omega)$ characterizes the purity of the heralded photon state as well as the degree of spectral decorrelations within the biphoton wave function.

The biphoton production rate p_2 and spectral correlations ξ_0 depend on four experimental parameters:

- pump beam waist – w_p
- pulse duration – τ_p
- crystal thickness – L
- crystal cut angle – θ_c

An analytical form of the biphoton wave function can be derived in the limiting case when the phase matching condition is satisfied for all relevant wave vectors and frequencies, i.e. $k_{pz} - k_{sz} - k_{iz} \equiv 0$. In this regime, the generation probability is maximized for the ratio of the collection mode and the pump beam waists equal to $w_f/w_p = 2$, and the photons are spectrally decorrelated when

$$\frac{c\tau_p}{w_p} = \sqrt{\frac{2}{3}} n_o(\omega_0)\alpha$$

These two observations serve as a starting point in the numerical optimization procedure.

MAIN RESULTS

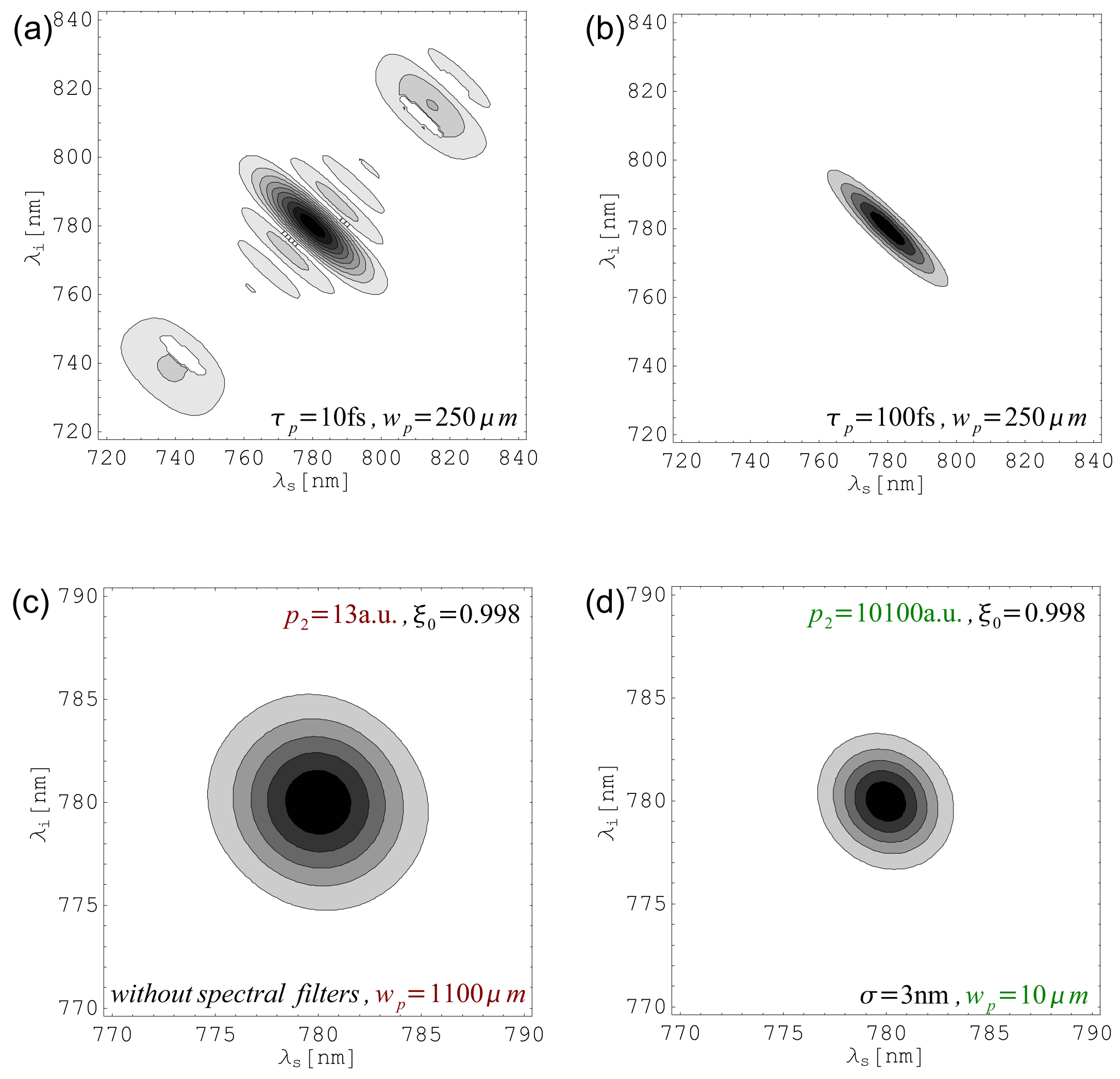

FIGURE 2: Absolute value of wave function of biphotons generated in type I nonlinear crystal of thickness $L = 1$mm and cut angle $\theta_c = 29.7°$, pumped by a pulse of $\tau_p = 100$fs duration and a beam waist $w_p = 155\mu$m. a) Pump pulse spectrum is broad, b) pump pulse spectral bandwidth comparable with the crystal phase matching bandwidth, c) pump pulse spectral bandwidth and beam width giving spectral decorrelation c) strongly focused pump beam combined with spectral filtering of biphotons.

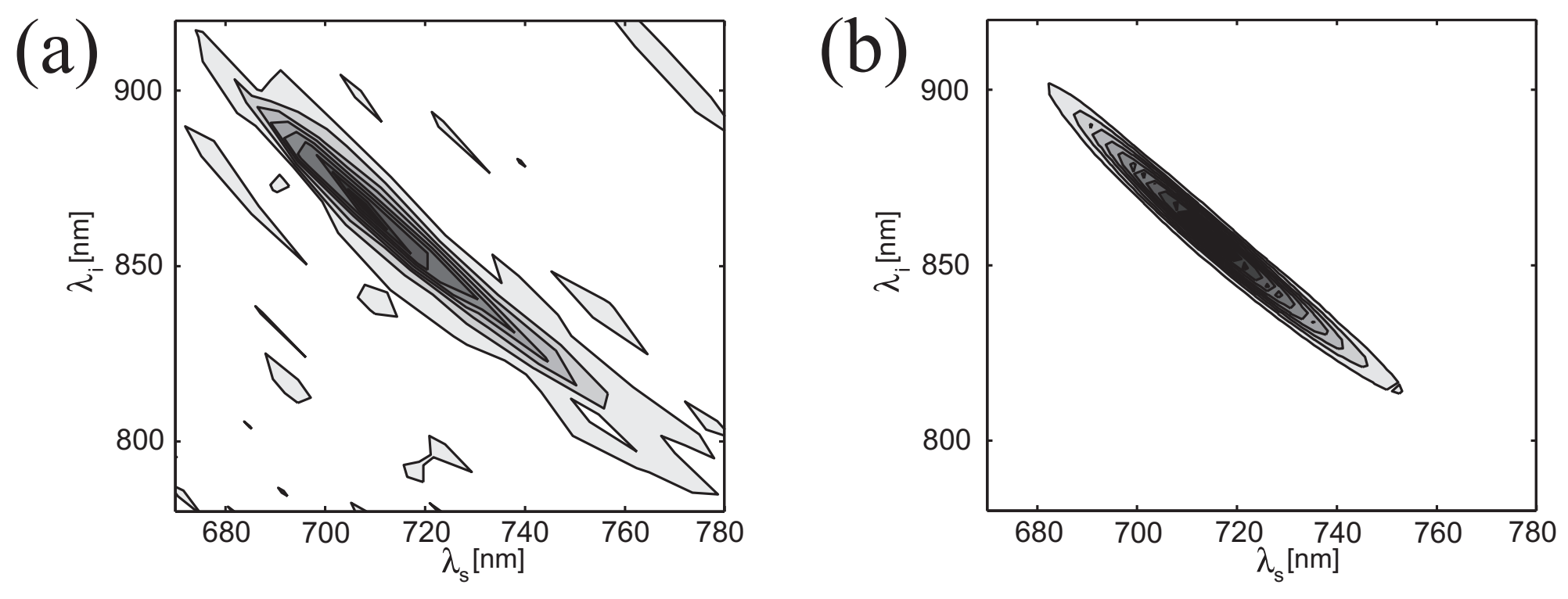

FIGURE 3: Contour plots of joint spectrum of photon pairs generated in type I crystal. Plot a) shows experimental outcomes from [2] for $L = 1$mm, $\theta_c = 29.7°$, $w_p = 155\mu$m, $\tau_p = 100$fs. Plot b) presents theoretical predictions based on our model [1].

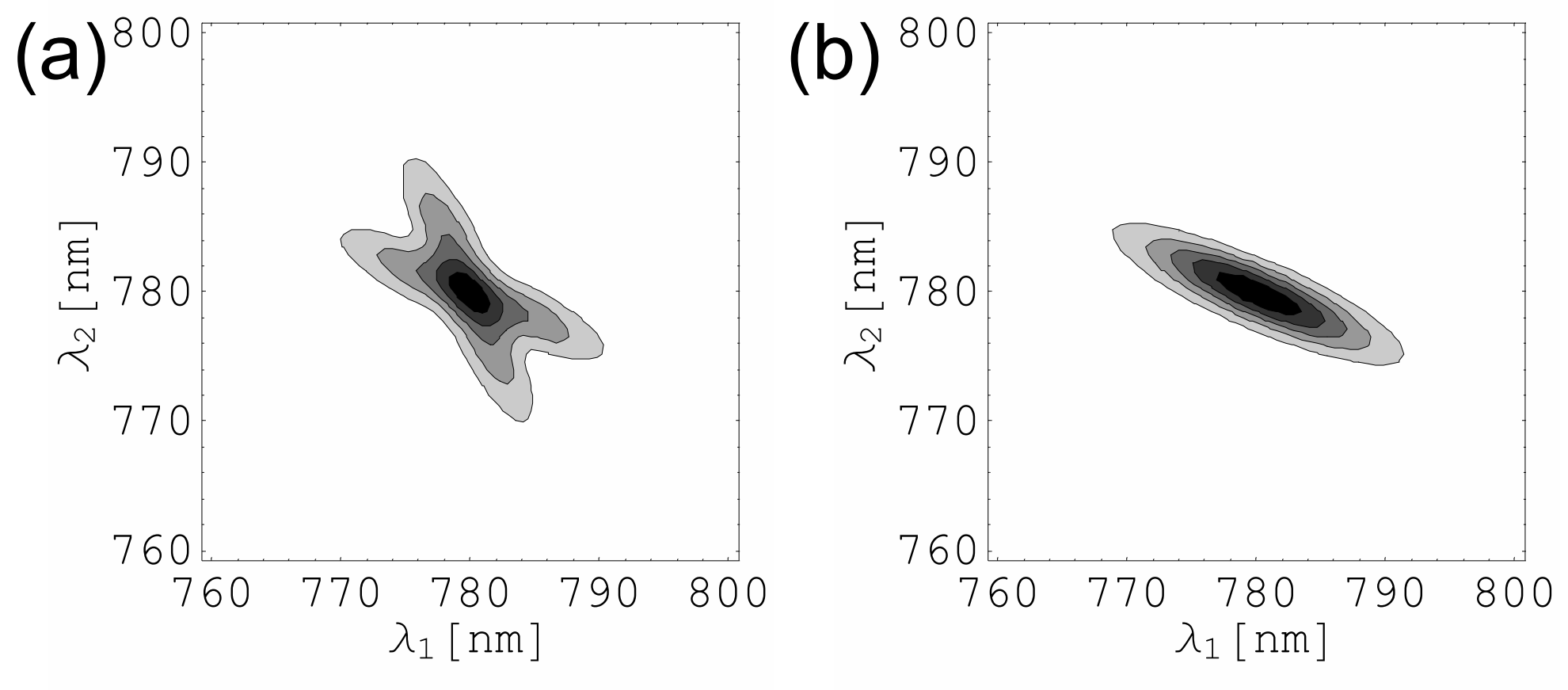

FIGURE 4: Contour plots of joint spectrum of photon pairs generated in type II crystal. Plots show theoretical predictions for nonlinear crystal of thickness $L = 2$mm and cut angle $\theta_c = 43.6°$ pumped by beam of $\tau_p = 100$fs pulse duration and $w_p = 50\mu$m waist. Polarizers orientations are a) $45°$ and $-45°$ b) $0°$ and $90°$. The calculated spectra are in good agreement with experimental results [3].

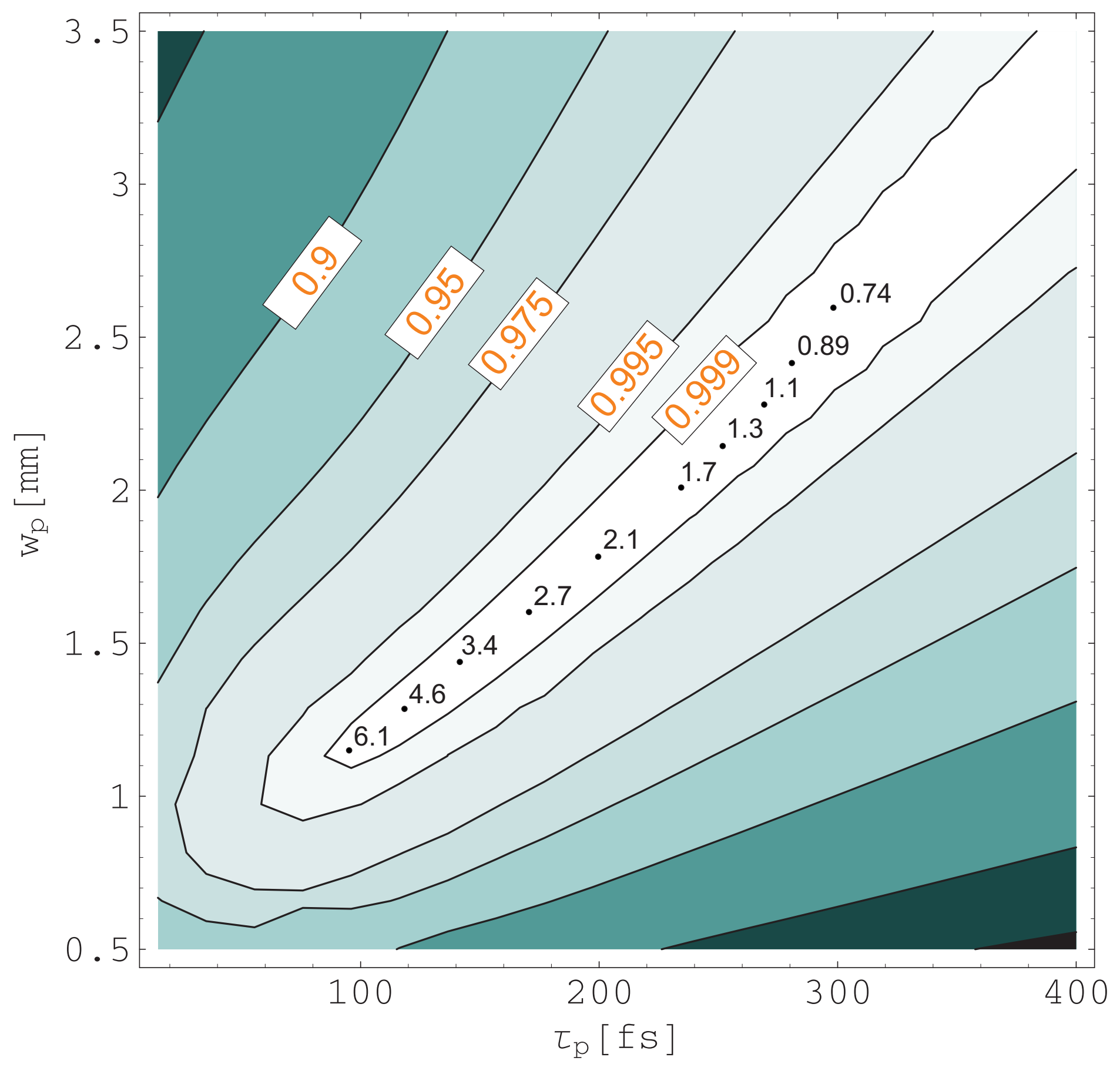

FIGURE 5: The contour plot shows how the first coefficient in the Schmidt decomposition ξ_0 depend on a beam waist w_p and pulse duration τ_p for fixed crystal parameters $L = 1$mm and $\theta = 30°$. Within the region $\xi_0 > 0.999$, biphoton production rates are given for values of (w_p, τ_p) marked with dots.

CONCLUSIONS

- We have presented a computationally efficient method of evaluating joint spectrum of photon pair generated in the process of SPDC.
- We derived an approximate analytical expression for setup parameters that optimize the brightness of the photon source and the spectral purity of heralded photons.
- The optimal way to construct a bright source of spectrally uncorrelated photon pairs is to strongly focus the pump beam and apply interference filters.
- The absence of spectral correlations allows one to prepare heralded photons in a spectrally pure state, which is necessary to achieve high-visibility multiphoton interference between independent sources.

REFERENCES

[1] P. Kolenderski, W. Wasilewski, K. Banaszek, to be published

[2] W. Wasilewski, P. Wasylczyk, P. Kolenderski, K. Banaszek, and Cz. Radzewicz, Opt. Lett. **31** (2006), 1130–1132.

[3] H. S. Poh, C. Y. Lum, I. Marcikic, A. Lames-Linares, C. Kirtsiefer, quantph/0608009

[4] A. Dragan, Phys. Rev. A, **70** (2004), 053814

[5] M. Rubin, D. N. Klyshko, Y. H. Shih, A. V. Sergienko, Phys. Rev. A, **50** (1994), 5122

[6] G. B. Boyd, D. A. Kleinman, J. Appl. Phys., **39** (1968), 3597

[7] C. K. Law, I. A. Walmsley, and J. H. Eberly, Phys. Rev. Lett. **84** (2000), 5304.

[8] A. B. U'Ren, C. Silberhorn, K. Banaszek, I. A. Walmsley, R. Erdmann, W. P. Grice, and M. G. Raymer, Las. Phys. **15** (2005), 1.