Spectral state of a single photon: the method of predicting and measurement

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Abstract

We propose and demonstrate a method for predicting and measuring the spectral density matrix of a single photon pulse coupled into single mode fiber. The experimental method is based on registering Hong-Ou-Mandel interference between the photon to be measured and a pair of attenuated and suitably delayed local oscillator pulses described by a known spectral amplitude. The density matrix is retrieved from a two-dimensional interferogram of coincidence counts. The method has been implemented for a type-I downconversion source, pumped by ultrashort laser pulses. The experimental results agree well with a theoretical model which assumes the exact phase matching function of the nonlinear crystal and the ultraviolet pump pulse shape computed based on the measured local oscillator spectral amplitude and phase.

Motivation

- What is the shape of a single photon?
- Complete characterization of a photon in a singlemode fiber - in particular its willingness to interfere
- Learn more about source

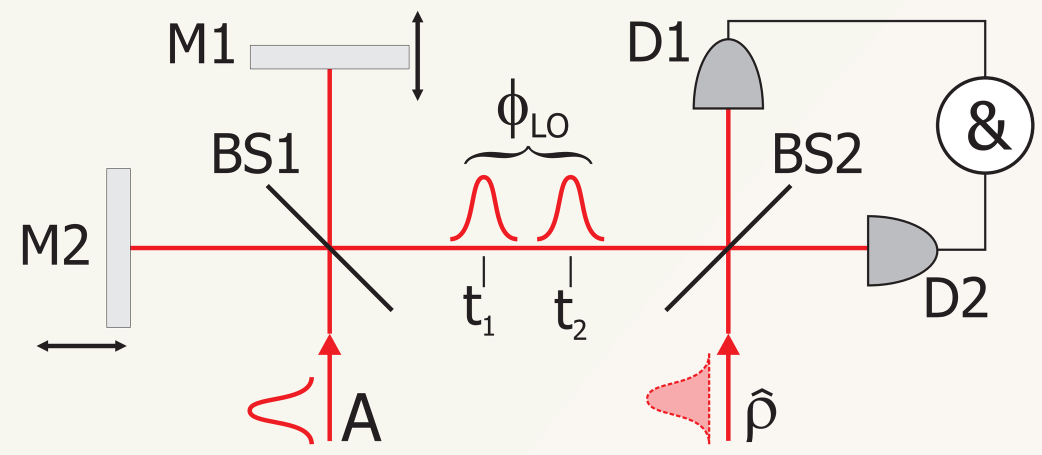
What do we measure?

When a single photon travels in a singlemode fiber, its spatial mode is well defined. For a fixed polarization its degree of freedom is the spectral one. The single photon density operator then is:

$$\hat{\rho} = \iint d\omega d\omega' \rho(\omega,\omega') \hat{a}^\dagger(\omega)|0\rangle\langle 0|\hat{a}(\omega')$$

Above, $\rho(\omega,\omega')$ is a density matrix in the spectral domain. This is the quantity that we measure.

Method



Scheme of the experiment. A weak laser pulse described by a spectral amplitude $A(\omega)$ enters a Michelson interferometer, comprising mirrors M1 and M2 and a 50/50 beamsplitter BS1, in which a double pulse of the local oscillator $\phi_{LO}(\omega)$ is produced. Then the LO interferes with an unknown photon described by the density operator on a 50/50 beamsplitter BS2 and may give rise to a coincidence click of the detectors D1 and D2.

When two photons interfere on a balanced beam-splitter BS2 a chance of registering coincidence p_c is a direct measure of an overlap Q of their spectral modes, $p_c = 1 - Q$. If one photon is described by unknown spectral density matrix, while the other comes form a local oscillator pulse of a known spectral amplitude $\phi_{LO}(\omega)$, the overlap equals:

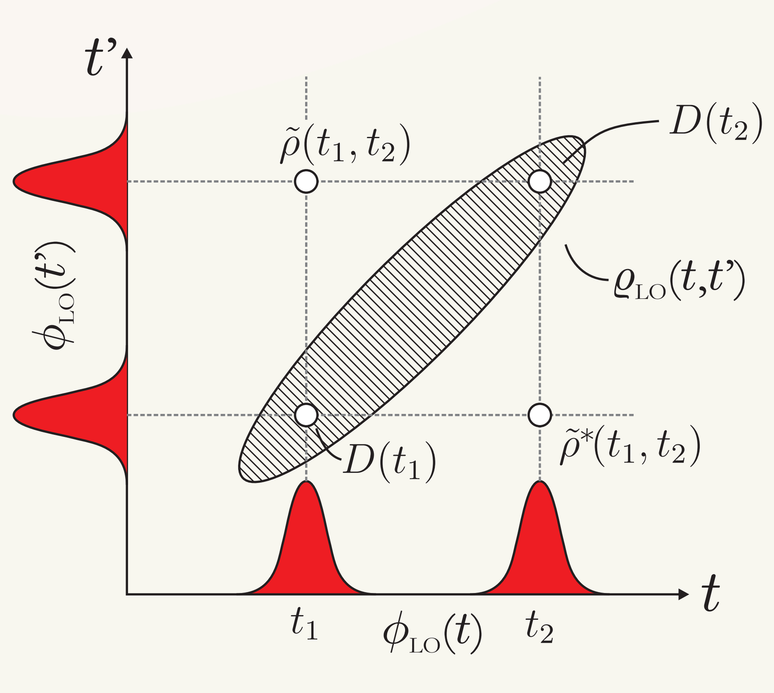
$$Q = \iint d\omega d\omega' \, \phi^*_{LO}(\omega)\rho(\omega,\omega')\phi_{LO}(\omega')$$

We find, that using double pulses at an adjustable delays as a LO is sufficient to retrieve complete density marix by inverting the above relation.

The spectral amplitude of a double pulse is:

$$\phi_{LO}(\omega) = A(\omega)\frac{\exp(-i\omega t_1) + \exp(-i\omega t_2)}{\sqrt{2S(t_2 - t_1)}}$$

where $S(t_2 - t_1)$ provides proper normalization, while $A(\omega)$ is the spectral amplitude of the reference laser pulses. With such an LO four terms contribute to the overlap Q, as easily pictured in the time domain:

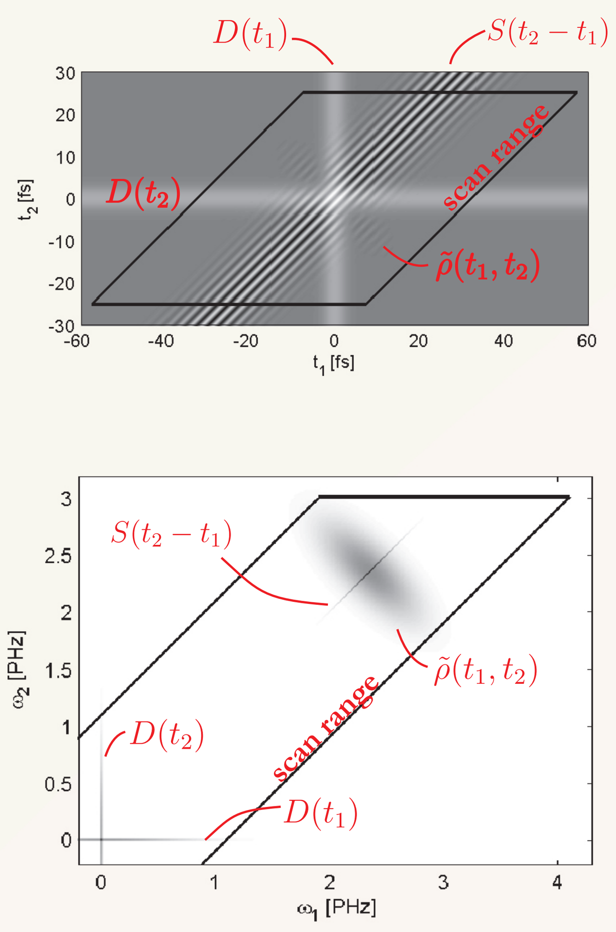


Substitution and expansion reveals:

$$Q(t_1,t_2) = \frac{D(t_1) + D(t_2) + 2\mathrm{Re}\tilde{\rho}(t_1,t_2)}{2S(t_2 - t_1)}$$

where $\tilde{\rho}(t_1,t_2)$ is a density matrix convolved with laser pulses:

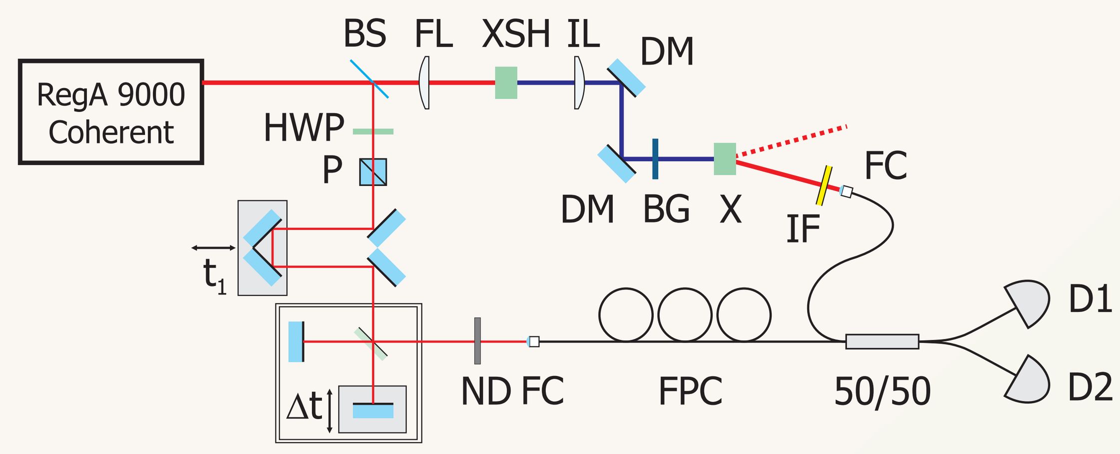
$$\tilde{\rho}(t_1,t_2) = \iint d\omega_1 d\omega_2 \, e^{i\omega_1 t_1 - i\omega_2 t_2} A^*(\omega_1)\rho(\omega_1,\omega_2)A(\omega_2)$$



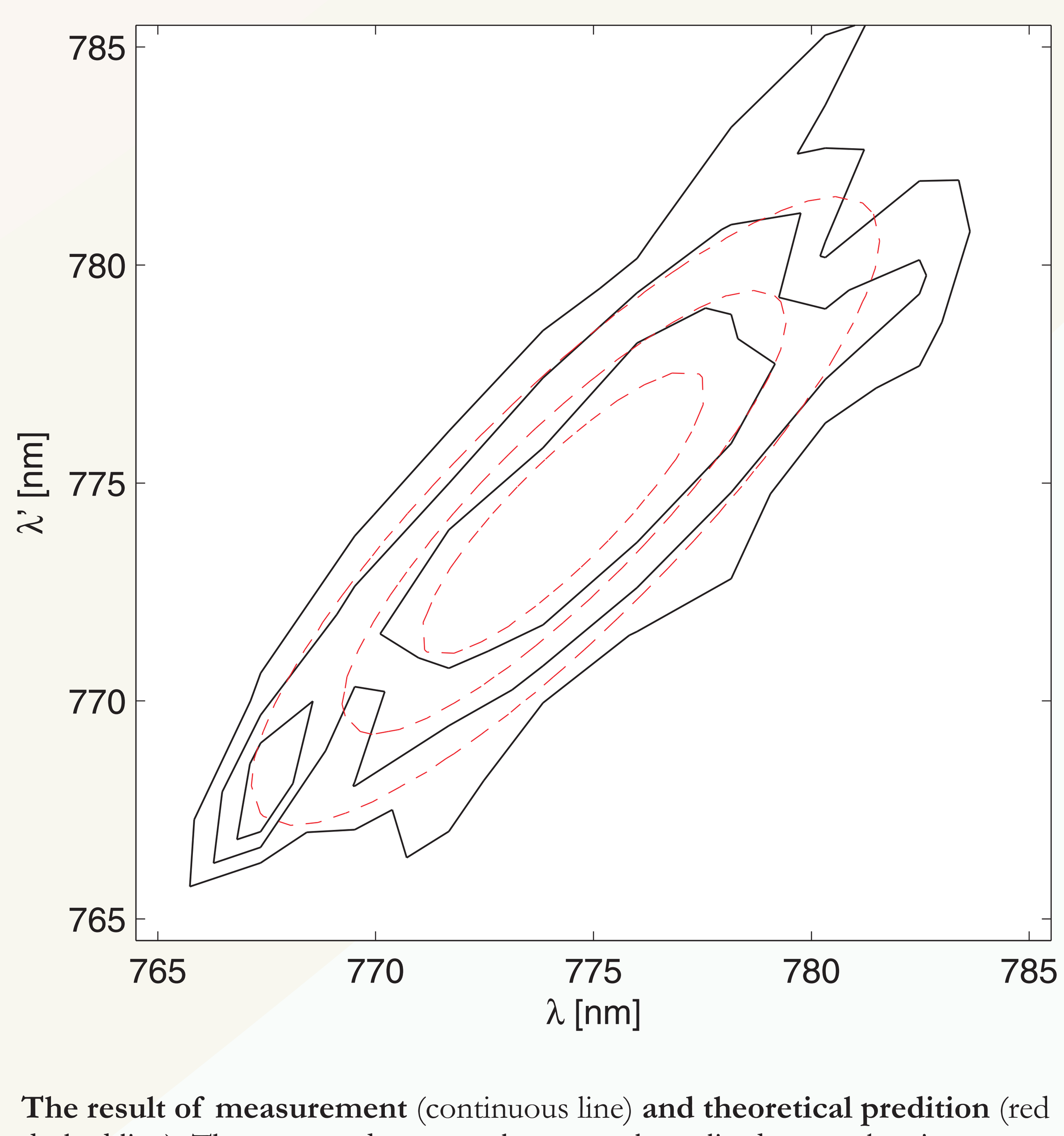
A typical **coincidence interferogram** $C(t_1,t_2)$ as a function of pulselets delays t_1 and t_2 and its fourier transform (below).

Retrieval of the spectral density matrix from collected coincidence counts $C(t_1,t_2)$ runs the following way: first the Fourier transform of $C(t_1,t_2)$ is computed. A region in the frequency space where contribution from the density matrix lies is separated. Next, a large contribution from classical interference between two pulselets is calculated and subsequently subtracted by measuring $C(t_1,t_2)$ for large delays when there is no overlap with the characterized photon. Finally, the density matrix is calculated by dividing the above result by the spectral amplitude of the master laser.

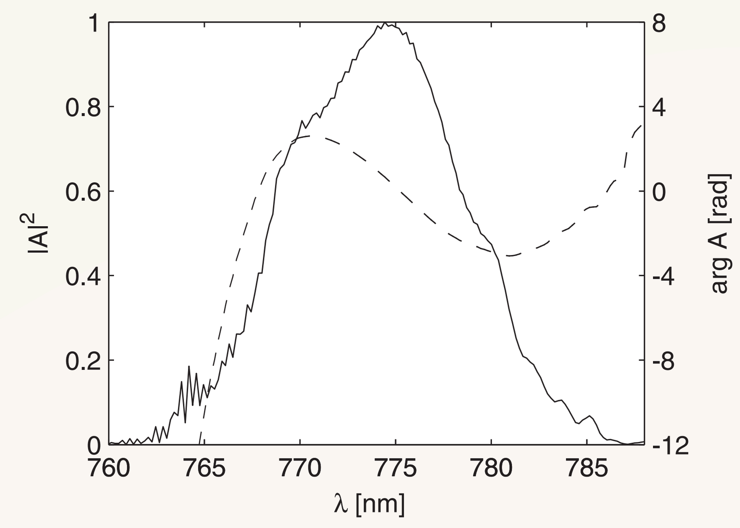
Experimental setup



The experimental setup. BS, 4% reflection beamsplitter; FL, 200mm focusing lens; XSH, 1mm thick type I BBO crystal for generation of the second harmonic; IL, 200mm imaging lens; DM dichroic mirrors; BG, blue glass filter; X, 1mm thick type I BBO downconversion crystal; IF, 10nm interference filter; FC, single mode fiber coupling stage, HWP, half waveplate; P, polarizer; ND, neutral density filter; FPC, fiber polarization controler; D1 and D2, single photon counting modules.

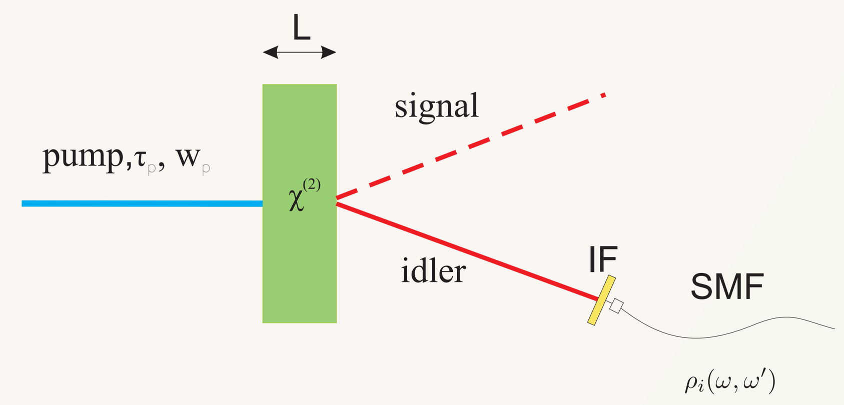


The result of measurement (continuous line) **and theoretical predition** (red dashed line). The measured pump pulse spectral amplitude was taken into account in numerical simulation.



Spectral intensity $|A(\omega)|^2$ (solid line) and phase $\arg A(\omega)$ (dashed line) of the master laser pulse retrieved using Frequency Resolved Optical Gating (FROG).

Model



Geometry of the photon source under consideration. We consider a $\chi^{(2)}$ nonlinear crystal of thickness L pumped by a pulsed gaussian beam of pulse duration τ_p and beam waist w_p. Generated photons in idler arm are coupled into SMF. Our object of interest is the reduced density matrix $\varrho_i(\omega,\omega')$ of single photon for spatial mode defined by SMF.

Steps[1]:

- We parameterize the biphotons with their frequencies ω_s, ω_i and transverse wave vectors k_s, k_i, where *s* and *i* correspond to the signal and the idler fields [2], respectively.
- The complete biphoton wave function parameterized with (ω_s, k_i, ω_i, k_i) involves an integral over the volume of the crystal of triple products of plane waves $\exp[i(k_{lz}z + k_l r)]$, $l = p,s,i$ where *p* corresponds to pump pulse.
- Integral over **r** yields conservation of the transverse momentum: $k_p = k_s + k_i$
- Analogously, integration over time gives $\omega_p = \omega_s + \omega_i$.
- The biphoton wave function when signal photon propagates in free space and the idler photon in SMF is obtained by performing projection onto spatial mode determined by the fiber.
- Longitudinal spatial frequencies k_{lz} are functions of ω_s, ω_i and k_s, k_i.
- The relevant range of k_i is restricted by the fiber spatial mode.
- The relevant range of k_s is restricted by the fiber spatial mode, because of momentum conservation.
- This justifies quadratic expansion of k_{lz} in k_i around central collection direction and in k_s around the phase maching direction; the dependence on ω_s and ω_i is kept exact, retaining all the orders of dispersion.
- Assuming the Gaussian shape of fiber and pump modes, all the integrals in transverse *xy* degrees of freedom can be evaluated analytically yielding the two-photon wave function $\psi(z;\mathbf{k}_{s\perp},\omega_s,\omega_i)$ generated by a slice (z,z+dz) of the crystal [3].
- The total generated wave function is thus given by a single integral over z:

$$\psi(\mathbf{k}_{s\perp},\omega_s,\omega_i) = \int dz \psi(z;\mathbf{k}_{s\perp},\omega_s,\omega_i)$$

- Tracing over signal photon degrees of freedom the spectral density matrix of idler photon reads:

$$\rho(\omega,\omega') = \int dz dz' \rho(z,z',\omega,\omega')$$

where

$$\rho(z,z',\omega,\omega') = \int d\mathbf{k}_{s\perp} d\omega_s \psi(z;\mathbf{k}_{s\perp},\omega_s,\omega)\psi^\star(z';\mathbf{k}_{s\perp},\omega_s,\omega')$$

Note that $\rho(z,z',\omega,\omega')$ is analytical and $\rho(\omega,\omega')$ is given by a double integral over the length of the crystal, thus the spectral density matrix can be efficiently evaluated by numerical means.

Conclusions

We proposed and demonstrated a method for measuring the spectral density matrix of a single photon component of the electromagnetic field in a singlemode fiber, which allows retrieving amplitude as well as phase of the density matrix. We applied this method to a downconversion-based source of single photons.

Additionally, we presented accurate and computationally efficient way of evaluating a spectral density matrix of single photon generated in a process of SPDC. Theoretically calculated ϱ agrees well with the measured one.

References

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[3] G. B. Boyd, D. A. Kleinman, J. Appl. Phys., **39** (1968) 3597