Computational Intelligence: Methods and Applications

Lecture 31

Combinatorial reasoning, or learning from partial observations.

Włodzisław Duch

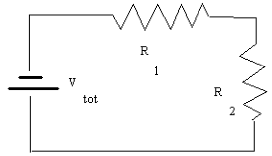
SCE, NTU, Singapore

Google: Duch

Learning from partial observations: intuitive thinking

A simple question in basic electronics: for a circuit on the picture, if R_2 increases, and R_1 and V_t are constant, what will happen with current I, and voltages V_1, V_2 ?

Trained student can answer such questions without making explicit calculations, AI expert system needs to write and solve equations by making many transformations.



How do we reason in such case?

We use intuition based on experience, partial observations of local changes. What is intuition? Just “look and see”?

R_2 increases so the total resistance should be higher, so the current I should decrease, and therefore V_1 will also decrease, and to keep the total voltage V_t constant V_2 should increase. No reference to equations.

Expert system approach

One way to answer the question would be to create an expert system with knowledge of basic laws of physics, able to analyze equation, transform them and solve them. A lot of programming and not easy.

Symbolic algebra packages (like Mathematica) may solve the problem in this way once we formulate it.

What useful knowledge do we have? Equations for partial relations:

Ohm’s law: V=I*R, may be applied to each resistor and total R:

$V_t=I*R_t$, $V_1=I*R_1$, and $V_2=I*R_2$

Kirchoff’s law: $V_t=V_1+V_2$

Adding resistances: $R_t=R_1+R_2$

Task: given information about R_2, R_1 and V_t find I, and voltages V_1, V_2

Calculate first R_t, then $I=V_t/R_t$, then V_1, and V_2.

But: change of the problem => new transformations needed.

Qualitative physics

What is the basic relation that we know?

Partial observation: A=f(B,C), either A=B*C, or A=B+C, and also $A^{-1}=B^{-1}+C^{-1}$ for parallel resistors.

How is the change of A correlated with changes of B and C?

In all cases A grows if B and C grow: there are 13 such true facts:

(A,B,C) = (+,+,+), (+,+,–), (+,–,+), (+,+,0), (+,0,+)
(0,+,–), (0,–,+), (0,0,0)
(–,–,–), (–,+,–), (–,–,+), (–,–,0), (–,0,–)

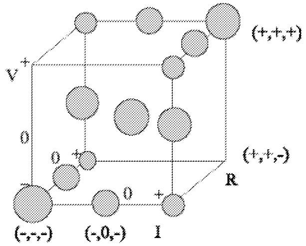
where +, 0, – means growing, staying constant and decreasing.

There are 14 false facts:

(A,B,C) = (+,–,0), (+,0,–), (+,0,0), (+,–,–)
(0,+,+), (0,+,0), (0,0,+), (0,0,–), (0,–,0), (0,–,–)
(–,+,+), (–,+,0), (–,0,+), (–,0,0)

Feature space representation

True facts are represented by local maxima of PDF, with more data for (+,+,+) than for (+,+,–) situation.



Amazing, but all 3-term formulas have identical representation in the feature space! 13 facts are true and 14 false.

Functional representation may be created using small Gaussians:

$F(\mathbf{X})=F(\mathrm{A,B,C})=\exp(-10*\|\mathbf{X}-(-1,-1,-1)\|^2) + \dots \exp(-10*\|\mathbf{X}-\mathbf{0}\|^2) + \dots$
$+ \exp(-10*\|\mathbf{X}-(+1,+1,+1)\|^2)$ (13 terms)

In the electric circuit example 7 variables are given, and all 5 laws that are applicable should be fulfilled.

How many facts are true in 7-D space?
There are 3^7=2187 possibilities: (–,–,–,–,–,–,–) to (+,+,+,+,+,+,+).
Only 111 combinations are consistent with all 5 laws!

Determining the truth intuitively

True facts correspond to inputs with non-zero PDFs.
Since 5 facts are true a product must be non-zero:

$$F(\mathbf{X}) = F(V_t, V_1, V_2, R, R_1, R_2, I) = \prod_{i=1}^{5} F_i(A_i, B_i, C_i)$$

There are 3^7=2187 input combinations,
but only 5% (111) of them are true, that is consistent with all constraints.

More constraints, more complex cases => less possibilities …
Here going from 3 to 7 variables gives reduction from 48% to 5%.

After learning qualitative properties of $F(A,B,C)$ relations, without any symbol manipulation just check if $F(\mathbf{X})>0$ to find true facts!

This is easy to do if you already know $\mathbf{X}$, but how to calculate $F(\mathbf{X})$ if some factors X_i are unknown?

Reasoning as search in the feature space

Question: knowing that R_2=+, R_1=0 and V_t =0,
how to determine changes in the remaining variables?

Find the values of the remaining variables that give
$F(V_t{=}0,V_1,V_2,R_t,R_1{=}0,R_2{=}+,I) > 0$

There may be just one set of such missing values, or many sets –in the case when nothing is known there are 111 such sets.

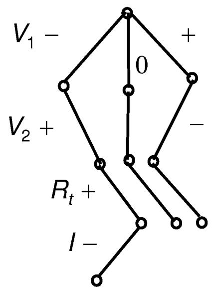
Human reasoning here is: assume that X_1 has some particular value, is it possible? If there is no immediate reason that is not possible, assume now that X_2 has some value.
Check $F(\mathbf{X})$ and some factors that have values for all variables may already give 0, so try another value for X_2.

For example, take the next unknown variable V_1 and assume that the unknown variable R_t = –, check: is it possible?

Search example

Since $F(V_t{=}0,V_1,V_2,R_t{=}-,R_1{=}0,R_2{=}+,I)=0$, then one of the laws is not fulfilled, so $R_t = -$ is not possible. It does not matter which law, the system “intuitively” answers: impossible! Is V_1=+ possible? Yes.

Try different values and create a search tree:



V_1=+, 0, – are all possible;

if V_1=– then V_2=–, 0 are not possible, so

V_2=+ is the only possibility;

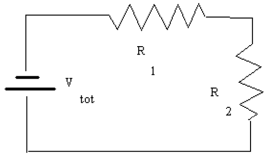
if V_1=0 then V_2=0 and if V_1=+ then V_2=–

This search tree has only one solution.

In general many solutions may be found.

Is this how you reason? We are usually a bit smarter: use heuristics!

Useful heuristics

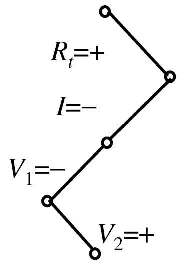


R_2=+, R_1=0 and V_t =0,

If a variable that we have considered first may take any value than it is not very informative.

Use variables that have greatest constraints!

If for some variable no value is possible than there is no solution.



If a variable takes only one value the search is greatly simplified: in our example R_t is the best variable since it may only grow; then I may only decrease, and that leads to a unique solution for two variables, V_1 and V_2.

A program based on this ideas may answer any question related to variable changes, and feasibility of constraints, for example, find all 111 possible situations that agree with all laws of physics.

Note that constraints may be soft! F(X) will measure overall agreement.

Learning from partial observations

If laws are know they may be presented as functions $F_m(X_{i1} ..X_{ik})$ that constraint possible solutions. If not, look for maxima of PDF in different subspaces and learn where these functions are approximately true.

All relations should be true at the same time, therefore we should take a product of all these functions and search for the maxima.

Use heuristic search, starting from most informative variables.

There may be several equivalent global maxima, and many smaller maxima that violate some constraints.

Each maximum corresponds to one possible solution.

This may be used for approximate constrained optimization or a search of acceptable subsets of feature values, etc.

Note: this is not yet in the textbooks, the only paper about it is:

Duch W & Diercksen GHF (1995) Feature Space Mapping as a universal adaptive system. Computer Physics Communications 87: 341-371